Distributed Runtime Monitoring

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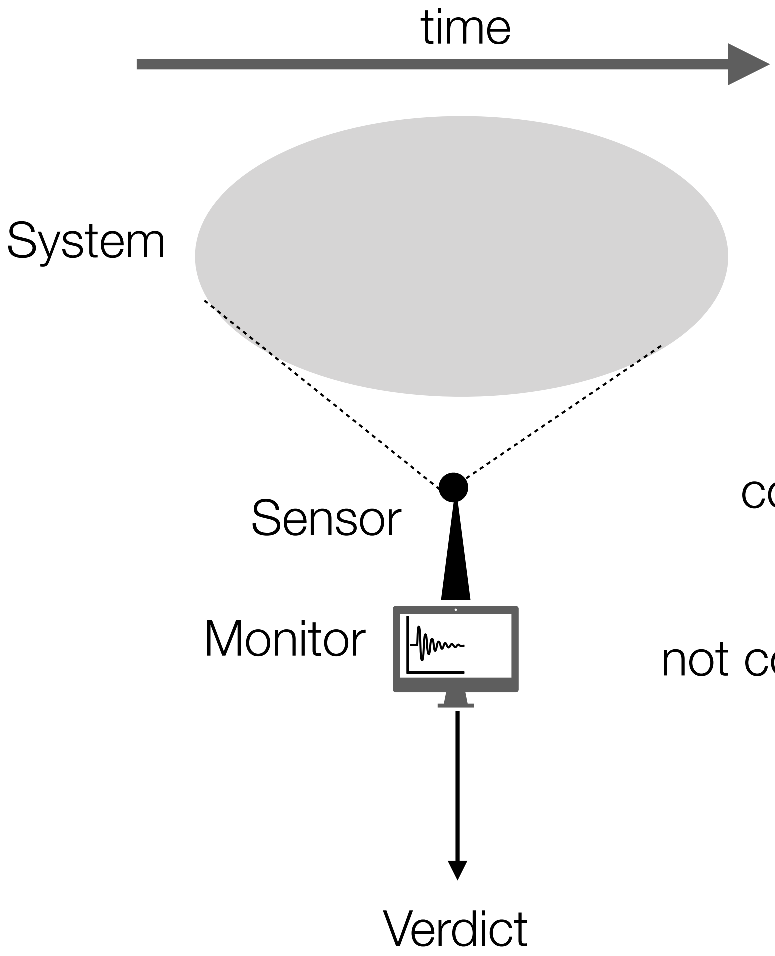
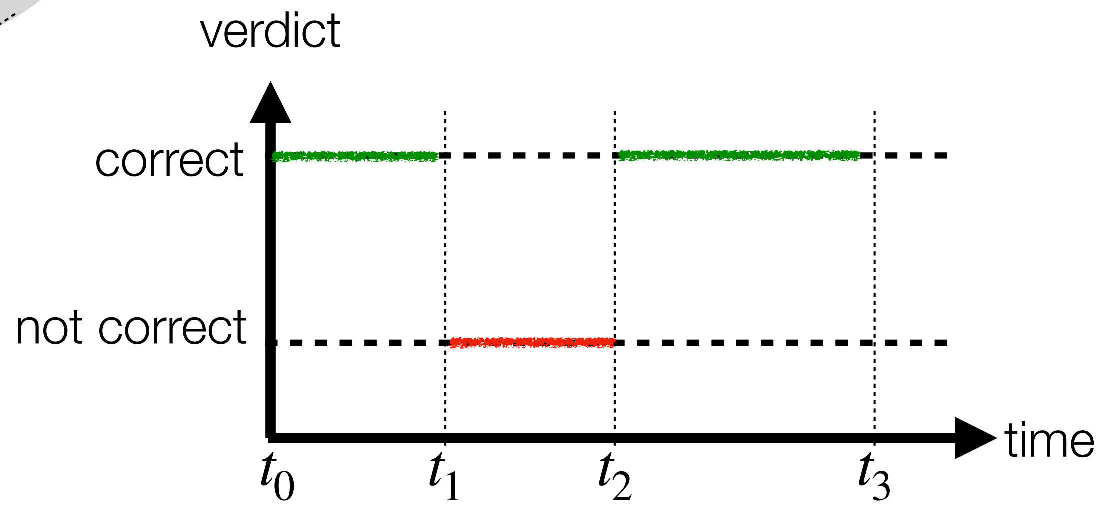
Runtime Monitoring





Runtime Monitoring Large Remote Systems

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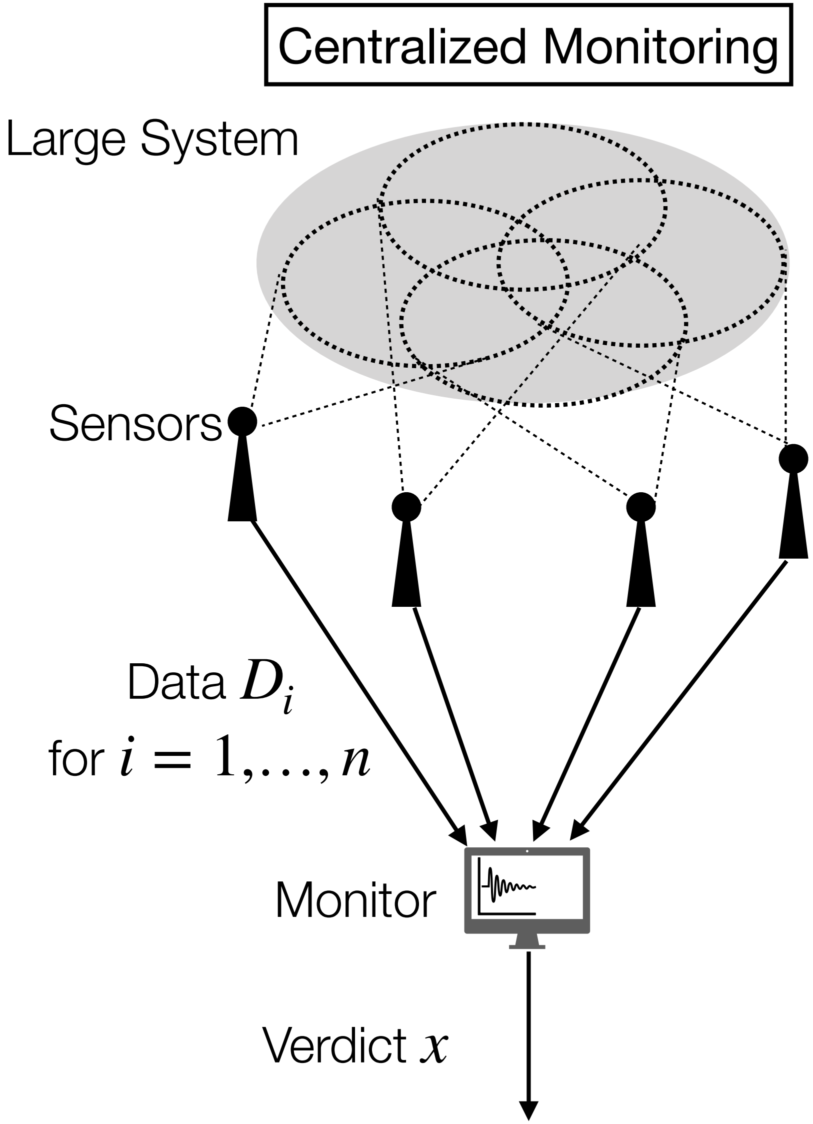

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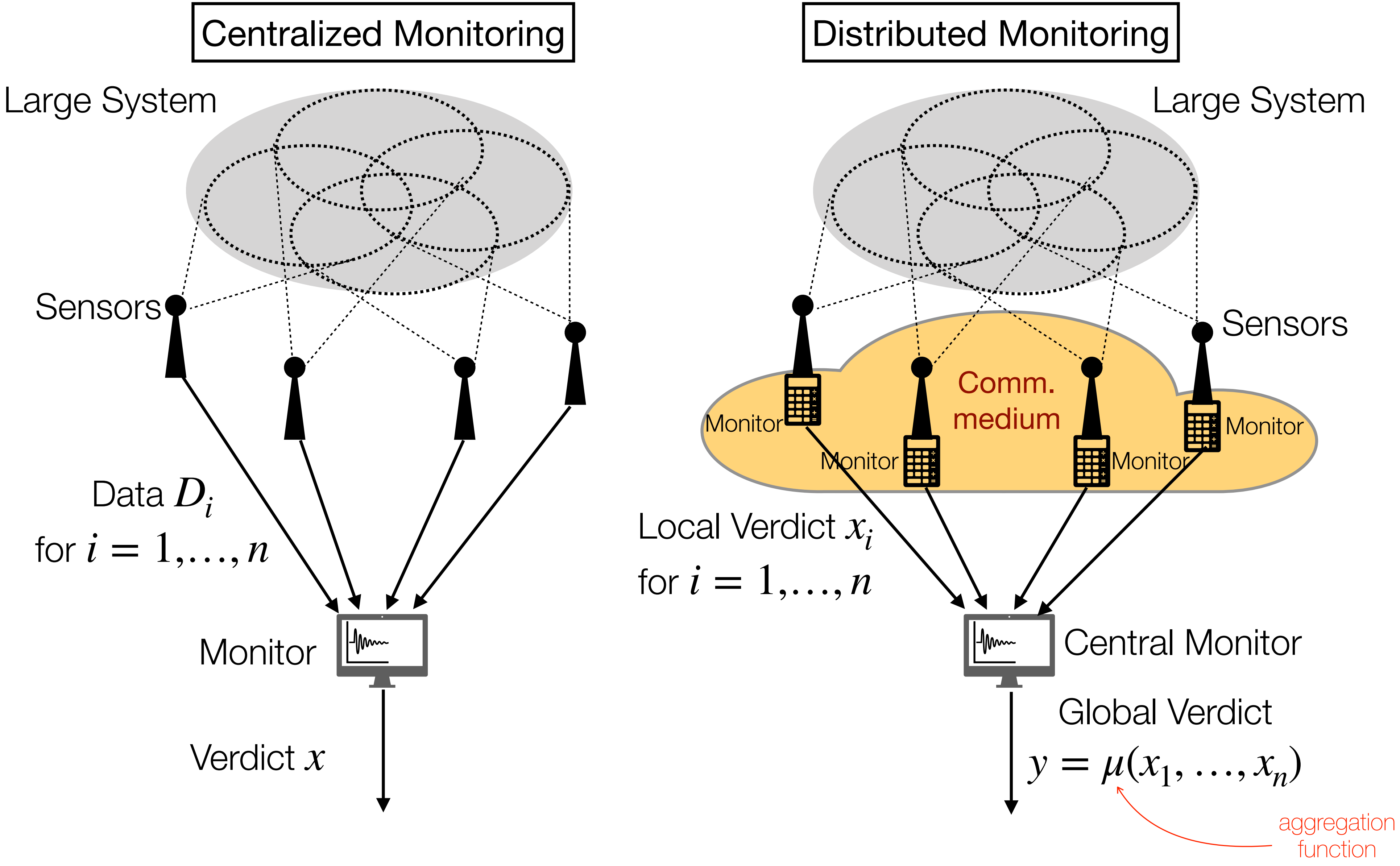

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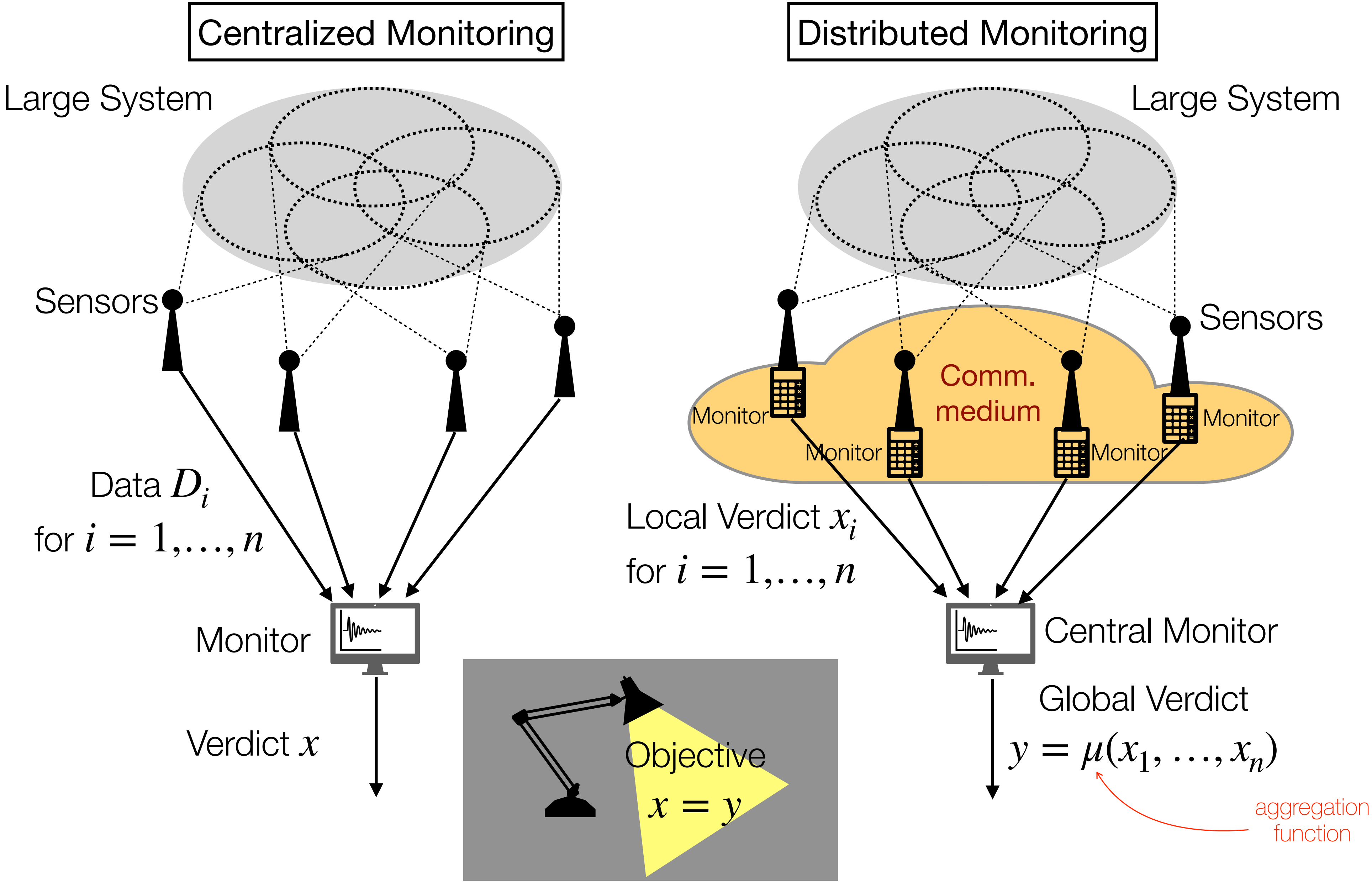

Linear Temporal Logic (LTL)

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Logical operators ¬ and ∨, and temporal operator X (*next*) and U (*until*):

- $[\sigma \models \mathsf{X}\varphi] \iff [\sigma^1 \models \varphi]$ where $\sigma^i = s_i s_{i+1} \ldots$
- $[\sigma \models \varphi \mathsf{U} \psi] \iff \exists i \geq 0 : ([\sigma^i \models \psi]) \wedge (\forall 0 \leq j < i, [\sigma^j \models \varphi])$

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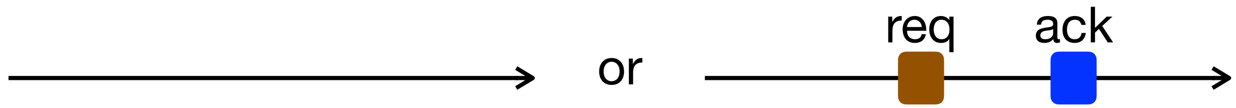
Enables to build other logical operators (∧, →, ↔, true, false) and other temporal operators, such as:

- F (*finally*): $\mathsf{F}\,\psi \equiv \mathsf{true}\,\mathsf{U}\,\psi$
- G (*globally*): $\mathsf{G}\,\psi \equiv \neg(\mathsf{F}\neg\psi)$
- R (*release*): $\psi\,\mathsf{R}\,\varphi \equiv \neg(\neg\psi\,\mathsf{U}\,\neg\varphi)$

Example: Req./Ack.

$$\varphi_{ra} = \mathsf{G}(\neg r \wedge \neg a) \vee \big((\neg a\,\mathsf{U}\,r) \wedge \mathsf{F}a\big)$$



- $\mathsf{G}(\neg a \wedge \neg r)$ = there are no req and no ack
- $(\neg a\,\mathsf{U}\,r) \wedge \mathsf{F}a$ = a req eventually occurs, not ack occur before that, and an ack must eventually occur.

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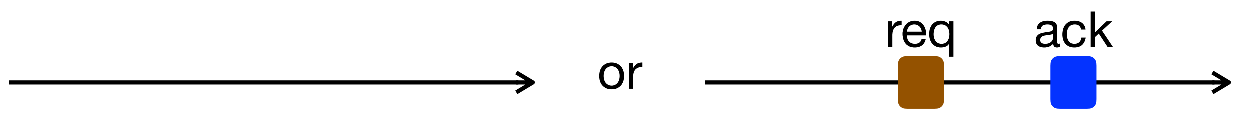

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$$\varphi_{ra2} = \Big(\mathsf{G}(\neg a_1 \wedge \neg r_1) \vee [(\neg a_1 \,\mathsf{U}\, r_1) \wedge \mathsf{F}a_1]\Big)$$

$$\wedge \Big(\mathsf{G}(\neg a_2 \wedge \neg r_2) \vee [(\neg a_2 \,\mathsf{U}\, r_2) \wedge \mathsf{F}a_2]\Big)$$

Finite LTL

Finite LTL (FLTL) is essentially LTL on finite traces $\alpha = s_0 s_1 \cdots s_t$

$$\left[\alpha \vDash_F \mathsf{N}\, \varphi\right] = \begin{cases} \left[\alpha^1 \vDash_F \varphi\right] & \text{if } \alpha^1 \neq \epsilon \\ \bot & \text{otherwise} \end{cases}$$

$$\left[\alpha \vDash_F \varphi \,\mathsf{U}\, \psi\right] = \begin{cases} \top & \text{if } \exists i \in \{0, \ldots, t\} : \Big(([\alpha^i \vDash_F \psi] = \top\ ) \\ & \quad \wedge (\forall j \in \{0, \ldots, i-1\}, [\alpha^j \vDash_F \varphi] = \top\ ) \Big) \\ \bot & \text{otherwise} \end{cases}$$

Multivalued Logic(s)

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3-valued LTL (LTL_3):

$$[\alpha \models_3 \varphi] = \begin{cases} \top & \text{if} \quad \forall \sigma \in \Sigma^\omega : \alpha\sigma \models \varphi \\ \bot & \text{if} \quad \forall \sigma \in \Sigma^\omega : \alpha\sigma \not\models \varphi \\ ? & \text{otherwise} \end{cases}$$

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4-valued LTL (RV-LTL):

$$[\alpha \models_4 \varphi] = \begin{cases} \top & \text{if} \quad [\alpha \models_3 \varphi] = \top \\ \bot & \text{if} \quad [\alpha \models_3 \varphi] = \bot \\ \top_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \ \wedge \ [\alpha \models_F \varphi] = \top \\ \bot_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \ \wedge \ [\alpha \models_F \varphi] = \bot \end{cases}$$

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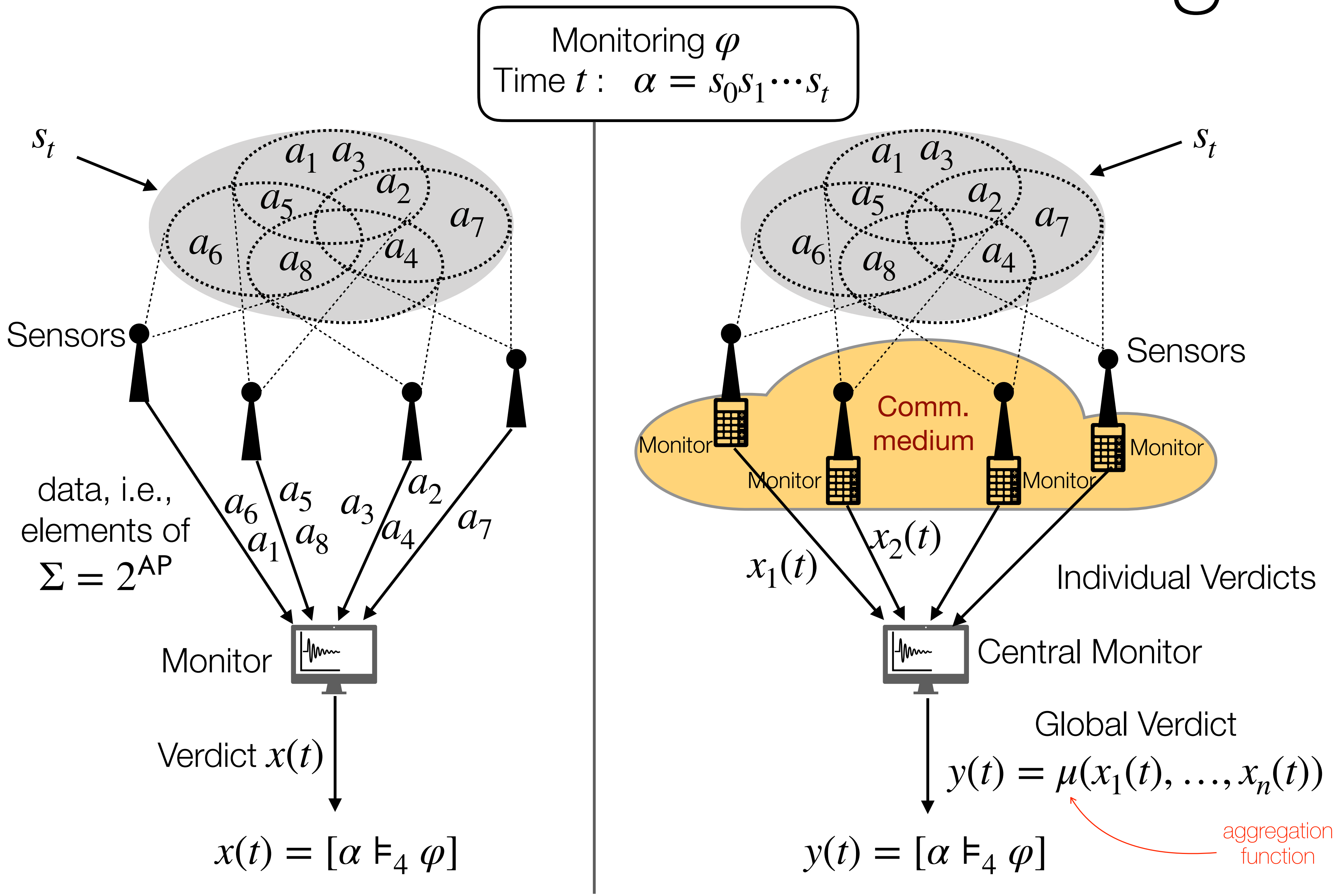
Runtime Verification LTL (RV-LTL)

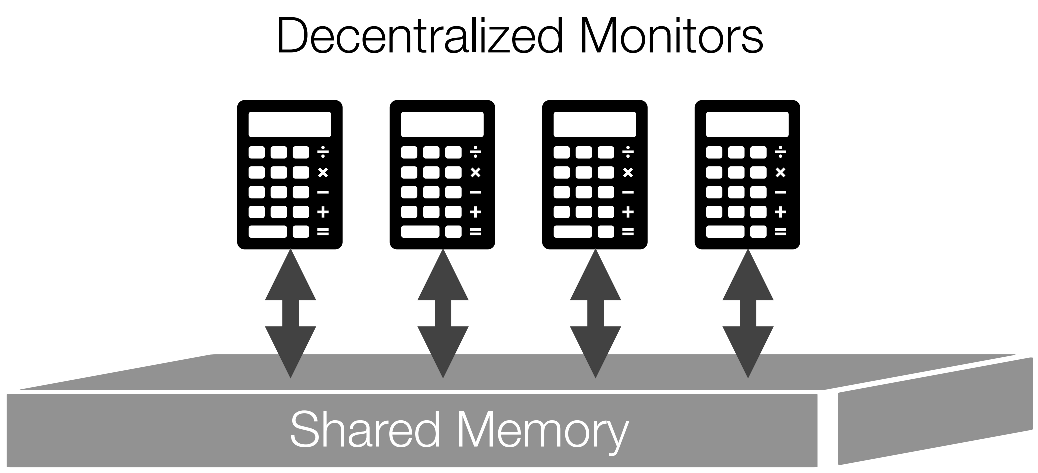
4-valued LTL (RV-LTL):

$$[\alpha \models_4 \varphi] = \begin{cases} \top & \text{if} \quad [\alpha \models_3 \varphi] = \top \\ \bot & \text{if} \quad [\alpha \models_3 \varphi] = \bot \\ \top_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \wedge [\alpha \models_F \varphi] = \top \\ \bot_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \wedge [\alpha \models_F \varphi] = \bot \end{cases}$$

Decentralized Monitoring



The Computational Model



Set of asynchronous crash-prone monitors

- Hypothesis 1: Shared memory with read/write accesses (we actually use IIS model)

- Hypothesis 2: Synchronization barrier between $\alpha' = s_0 s_1 \cdots s_{t-1}$ and $\alpha = s_0 s_1 \cdots s_t = \alpha' s_t$ for all $t \geq 1$

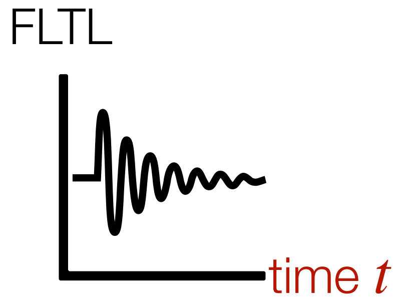
Monitors Get Partial Information

- Partial trace $\alpha = s_0 s_1 \cdots s_t$ where $\alpha' = s_0 s_1 \cdots s_{t-1}$ is fixed
- Monitors examine $s_t = \{a_1, \ldots, a_k\}$ for deciding $[\alpha \models_4 \varphi]$
- $w_i = \text{view}(p_i),\ i = 1, \ldots, n$, after some communication
- It may be the case that
  - $[\alpha' w_i \models_F \varphi] = \top$ for some $i \in [n]$
  - $[\alpha' w_j \models_F \varphi] = \bot$ for some $j \neq i$

Evaluation of φ Evolves Across both Time and Space

partial trace α

s_0 s_1 …… s_{t-1} s_t



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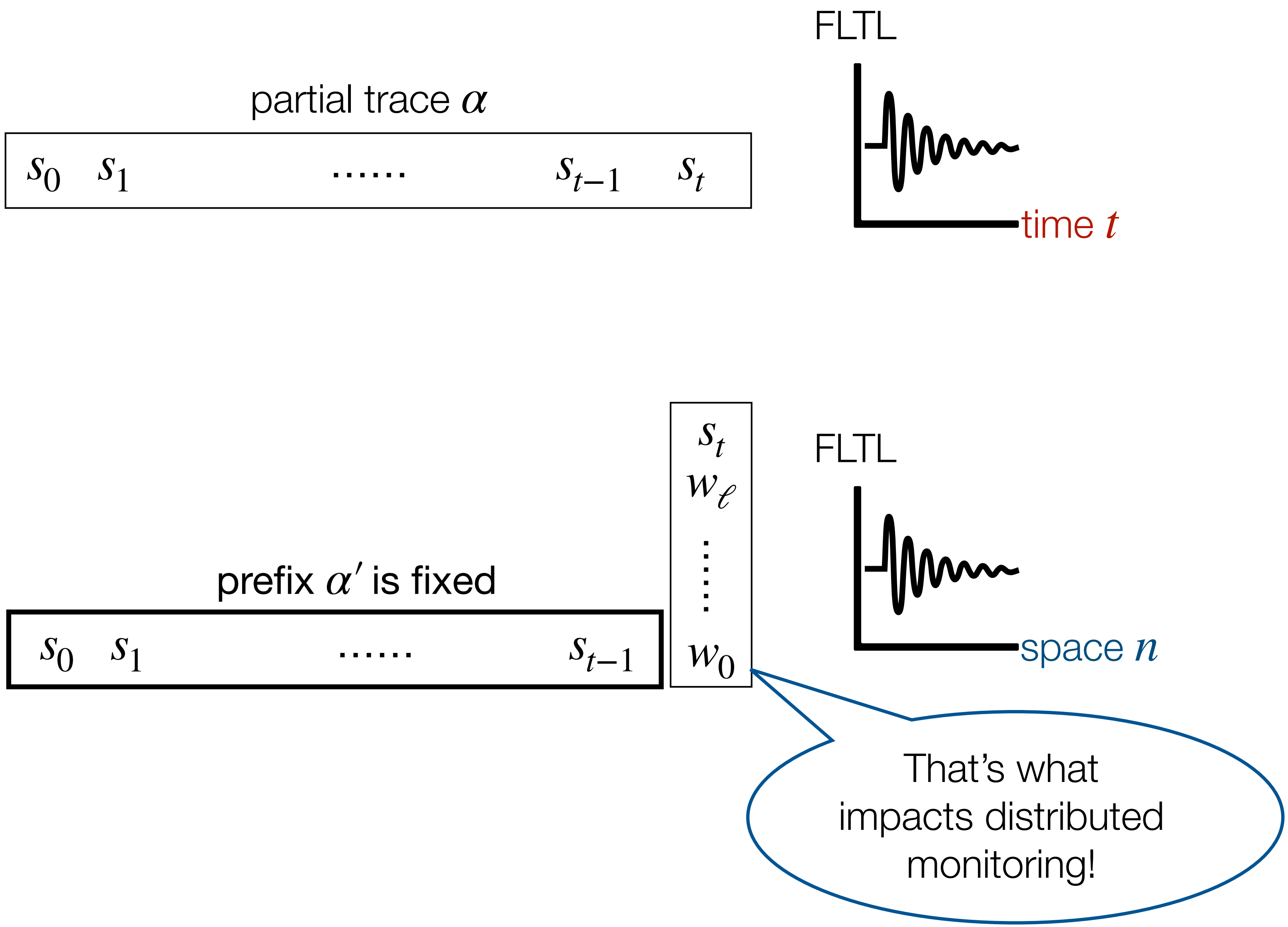

Alternation Number

- The *alternation number* of an LTL formula φ with respect to a finite partial trace $\alpha = \alpha' s$, denoted by $AN(\varphi, \alpha)$, is the maximum integer $k \geq 0$ such that there exists a sequence of partial states (i.e., views) $w_0, \cdots, w_k$ with $w_0 = \emptyset$, $w_k = s$, and, for every $i \in \{0, \ldots, k-1\}$,

$$\left(w_i \subsetneq w_{i+1}\right) \quad \wedge \quad \left([\alpha' w_i \models_F \varphi] \neq [\alpha' w_{i+1} \models_F \varphi]\right)$$

- The alternation number of an LTL formula φ is

$$AN(\varphi) = \max \left\{AN(\varphi, \alpha) \mid \alpha \in \Sigma^{\star}\right\}$$

- Remark: $AN(\varphi) \leq |\mathsf{AP}|$

Example

$$\varphi_{ra2} = \Big(\mathsf{G}(\neg a_1 \wedge \neg r_1) \vee [(\neg a_1 \,\mathsf{U}\, r_1) \wedge \mathsf{F} a_1]\Big)$$

$$\wedge \Big(\mathsf{G}(\neg a_2 \wedge \neg r_2) \vee [(\neg a_2 \,\mathsf{U}\, r_2) \wedge \mathsf{F} a_2]\Big)$$

$s_0 = \{r_1, a_1, r_2, a_2\}$

- $w_0 = \emptyset$ ➜ $[w_0 \models_F \varphi_{ra2}] = \top$
- $w_1 = \{r_1\}$ ➜ $[w_1 \models_F \varphi_{ra2}] = \bot$
- $w_2 = \{r_1, a_1\}$ ➜ $[w_2 \models_F \varphi_{ra2}] = \top$
- $w_3 = \{r_1, a_1, r_2\}$ ➜ $[w_3 \models_F \varphi_{ra2}] = \bot$
- $w_4 = \{r_1, a_1, r_2, a_2\} = s_0$ ➜ $[w_4 \models_F \varphi_{ra2}] = \top$

$$AN(\varphi_{ra2}) = 4$$

Results

Theorem For every $k \geq 0$, there exists an LTL formula φ with $AN(\varphi) = k$ such that runtime verifying φ using distributed monitors requires a verdict set V with $|V| \geq k + 1$.

Theorem For every $k \geq 0$, and for every LTL formula φ with $AN(\varphi) = k$, there is distributed monitors that correctly monitor φ using verdict set

$$\mathbb{B}_{2\lceil k/2 \rceil + 4} = \{ \bot , \top , \bot_0 , \top_0 , \ldots, \bot_{\lceil k/2 \rceil} , \top_{\lceil k/2 \rceil} \}$$

Distributed LTL

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Let $\alpha = \alpha' s$ be a finite partial trace

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$$
\left[\alpha \vDash_D \varphi\right] = \begin{cases}
\top & \text{if} \quad [\alpha \vDash_4 \varphi] = \top \\
\bot & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot \\
\top_0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \top_p \;\wedge\; (\forall w \subset s : [\alpha' w \vDash_D \varphi] = \top_0\,) \\
\bot_0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot_p \;\wedge\; (\forall w \subset s : [\alpha' w \vDash_D \varphi] = \bot_0\,) \\
\top_i\,, i > 0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \top_p \;\wedge\; (\,\exists w \subset s : [\alpha' w \vDash_D \varphi] = \bot_{i-1}\,) \\
 & \quad\quad \wedge\; (\forall w \subset s, \exists j < i : [\alpha' w \vDash_D \varphi] \in \{\, \top_j\,, \bot_j \,\} \cup \{\, \top_i \,\}) \\
\bot_i\; i > 0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot_p \;\wedge\; (\,\exists w \subset s : [\alpha' w \vDash_D \varphi] = \top_{i-1}\,) \\
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\end{cases}
$$

Distributed LTL

Let $\alpha = \alpha' s$ be a finite partial trace

$$[\alpha \vDash_D \varphi] = \begin{cases} \top & \text{if} \quad [\alpha \vDash_4 \varphi] = \top \\ \bot & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot \\ \top_0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \top_p \ \wedge \ (\forall w \subset s : [\alpha' w \vDash_D \varphi] = \top_0 \ ) \\ \bot_0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot_p \ \wedge \ (\forall w \subset s : [\alpha' w \vDash_D \varphi] = \bot_0 \ ) \\ \top_i \, , i > 0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \top_p \ \wedge \ (\ \exists w \subset s : [\alpha' w \vDash_D \varphi] = \bot_{i-1} \ ) \\ & \quad \wedge \ (\forall w \subset s, \exists j < i : [\alpha' w \vDash_D \varphi] \in \{ \ \top_j \, , \bot_j \ \} \cup \{ \ \top_i \ \}) \\ \bot_i \ i > 0 & \text{if} \quad [\alpha \vDash_4 \varphi] = \bot_p \ \wedge \ (\ \exists w \subset s : [\alpha' w \vDash_D \varphi] = \top_{i-1} \ ) \\ & \quad \wedge \ (\forall w \subset s, \exists j < i : [\alpha' w \vDash_D \varphi] \in \{ \ \top_j \, , \bot_j \ \} \cup \{ \ \bot_i \ \}) \end{cases}$$

$$AN(\varphi, \alpha) = \begin{cases} 0 & \text{if} \quad [\alpha \vDash_D \varphi] \in \{ \ \top \, , \bot \ \} \\ k & \text{if} \quad [\alpha \vDash_D \varphi] \in \{ \ \bot_k \, , \top_k \ \} \end{cases}$$

Reducing #Logical Values

DLTL^{+}

$\bot_0 < \top_0 < \bot_1 < \top_1 < \ldots < \top_{i-1} < \bot_i < \top_i < \bot_{i+1} < \ldots$

DLTL^{-}

$\top_0 < \bot_0 < \top_1 < \bot_1 < \ldots < \bot_{i-1} < \top_i < \bot_i < \top_{i+1} < \ldots$

Conclusion and Open Problems

- **Proof of concept:** Decentralized runtime monitoring of φ can be done, with verdicts in $\text{LTL}_{AN(\varphi)+O(1)}$

- **Conjecture:** For every φ, distributed monitoring of φ requires $AN(\varphi)$ different values.

- **Next step:** Getting rid of the synchronization barrier between $\alpha' = s_0 s_1 \ldots s_{t-1}$ and $\alpha = \alpha' s_t = s_0 s_1 \ldots s_{t-1} s_t$

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Grazie